Closing Wed night: HW_1A, 1B, 1C
Check out the first newsletter for homework hints, review sheets and old exam practice.

4.9 Antiderivatives

Def'n: If g(x) = f'(x), then we say

g(x) = "the derivative of f(x)", and

f(x) = "an antiderivative of g(x)"

Example:

Give an antiderivative of

$$g(x) = x^3 + 5$$

Examples (you do):

Find the general antiderivative of

1. $f(x) = x^6 - 3$
2. $g(x) = \cos(x) + \frac{1}{x} + e^x + \frac{1}{1+x^2}$
3. $h(x) = \frac{5}{\sqrt{x}} + \sqrt[3]{x^2}$
4. $r(x) = \frac{x-3x^2}{x^3}$

Initial Conditions: There is no way to know what "C" is unless we are given additional information about the antiderivative. Such information is called an **initial condition**.

Example: $f'(x) = e^x + 4x$ and

$f(0) = 5$

Find $f(x)$.

Example: $f''(x) = 15\sqrt{x}$, and

$$f(1) = 0, f(4) = 1$$

Find $f(x)$.

Example:

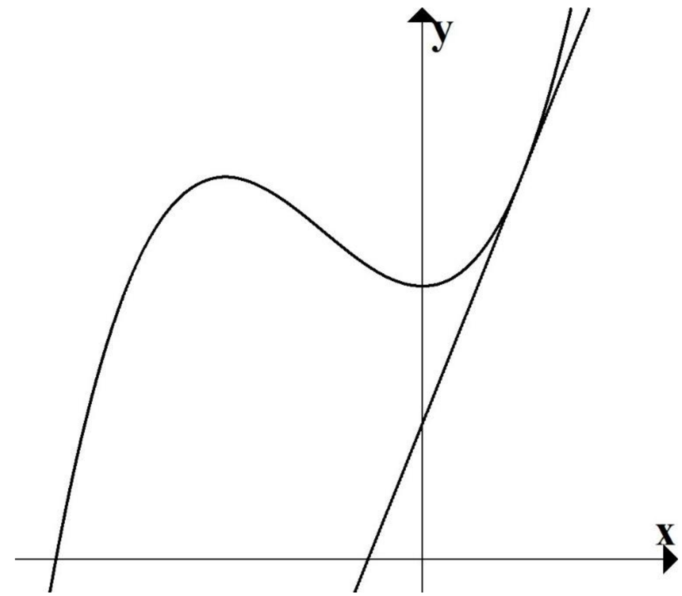
Ron *steps off* the 10 meter high dive at his local pool. Find a formula for his height above the water.

(Assume his acceleration is a constant 9.8 m/s^2 downward)

5.1 Defining Area

Calculus is based on limiting processes that "approach" the exact answer to a rate question.

Calc. I Visual:



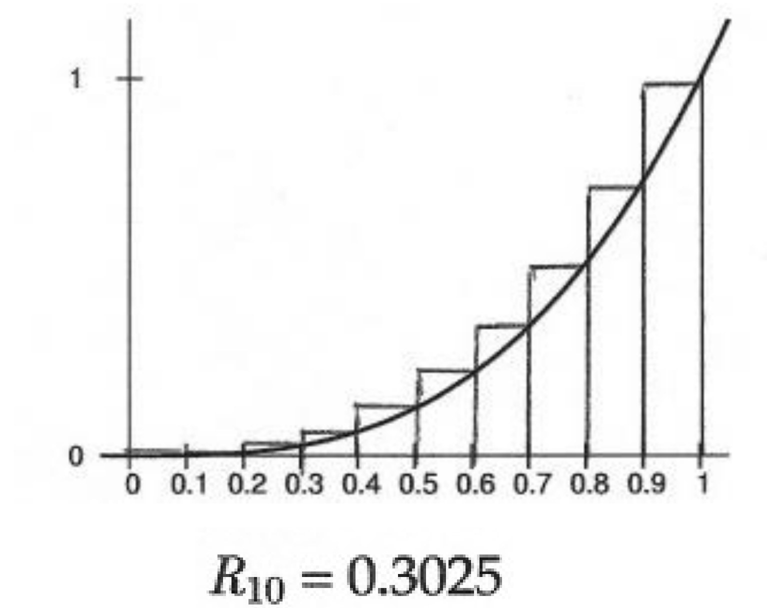
In Calculus I, you defined

f'(x) = `slope of the tangent at *x*'

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

In Calculus II, we will see that antiderivatives are related to the area `under' a graph

Calc. II Visual:



$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i{}^*)\Delta x$$

Riemann sums set up:

We are going to build a procedure to get better and better approximations of the area "under" $f(x)$.

1. Break into *n* equal subintervals.

$$\Delta x = \frac{b-a}{n} \text{ and } x_i = a + i\Delta x$$

2. Draw *n* rectangles; use function.
   Area of each rectangle =
   (height)(width) $= f(x_i^*)\Delta x$

3. Add up rectangle areas.

Example:

Approximate the area under $f(x) = x^3$ from x = 0 to x = 1 using n = 3 subdivisions and *right-endpoints* to find the height.

I did this again with 100 subdivisions, then 1000, then 10000. Here is the summary of my findings:

n	R_n	L_n
4	0.390625	0.140625
5	0.36	0.16
10	0.3025	0.2025
100	0.255025	0.245025
1000	0.25050025	0.24950025
10000	0.2499500025	0.2500500025

Pattern:

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}, \qquad x_i = 0 + i\frac{1}{n} = \frac{i}{n}$$

Pattern for each Rectangle

$$\text{Height} = f(x_i) = x_i^3 = \left(\frac{i}{n}\right)^3$$

$$\text{Area} = f(x_i)\Delta x = x_i^3 \Delta x = \left(\frac{i}{n}\right)^3 \frac{1}{n}$$

Adding up the area of each rectangle

$$\text{Sum} = \sum_{i=1}^{n} x_i^3 \Delta x = \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n}$$

$$\text{Exact Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n}$$

Definition of the Definite Integral

We define the exact area "under" f(x) from x = a to x = b curve to be

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x,$$

where $\Delta x = \frac{b-a}{n}$ and

$$x_i = a + i\Delta x.$$

We call this the definite integral of f(x) from x = a to x = b, and we write

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

Example: Write down this definition for the function $f(x) = \sqrt{x}$ on the interval $x = 5$ to $x = 7$.